

ELLIPTIC INTEGRAL OF THE THIRD KIND $\Pi(n; \varphi | \alpha)$

Table 17.9

$$\Pi(n; \varphi | \alpha) = \int_0^\varphi (1 - n \sin^2 \theta)^{-1} [1 - \sin^2 \alpha \sin^2 \theta]^{-\frac{1}{2}} d\theta$$

n	$\alpha \setminus \varphi$	0°	15°	30°	45°	60°	75°	90°
0.0	0°	0	0.26180	0.52360	0.78540	1.04720	1.30900	1.57080
0.0	15	0	0.26200	0.52513	0.79025	1.05774	1.32733	1.59814
0.0	30	0	0.26254	0.52943	0.80437	1.08955	1.38457	1.68575
0.0	45	0	0.26330	0.53562	0.82602	1.14243	1.48788	1.85407
0.0	60	0	0.26406	0.54223	0.85122	1.21260	1.64918	2.15651
0.0	75	0	0.26463	0.54736	0.87270	1.28371	1.87145	2.76806
0.0	90	0	0.26484	0.54931	0.88137	1.31696	2.02759	∞
0.1	0	0	0.26239	0.52820	0.80013	1.07949	1.36560	1.65576
0.1	15	0	0.26259	0.52975	0.80514	1.09058	1.38520	1.68536
0.1	30	0	0.26314	0.53412	0.81972	1.12405	1.44649	1.78030
0.1	45	0	0.26390	0.54041	0.84210	1.17980	1.55739	1.96326
0.1	60	0	0.26467	0.54712	0.86817	1.25393	1.73121	2.29355
0.1	75	0	0.26524	0.55234	0.89040	1.32926	1.97204	2.96601
0.1	90	0	0.26545	0.55431	0.89939	1.36454	2.14201	∞
0.2	0	0	0.26299	0.53294	0.81586	1.11534	1.43078	1.75620
0.2	15	0	0.26319	0.53452	0.82104	1.12705	1.45187	1.78850
0.2	30	0	0.26374	0.53896	0.83612	1.16241	1.51792	1.89229
0.2	45	0	0.26450	0.54535	0.85928	1.22139	1.63775	2.09296
0.2	60	0	0.26527	0.55217	0.88629	1.30003	1.82643	2.45715
0.2	75	0	0.26585	0.55747	0.90934	1.38016	2.08942	3.20448
0.2	90	0	0.26606	0.55948	0.91867	1.41777	2.27604	∞
0.3	0	0	0.26359	0.53784	0.83271	1.15551	1.50701	1.87746
0.3	15	0	0.26379	0.53945	0.83808	1.16791	1.52988	1.91309
0.3	30	0	0.26434	0.54396	0.85370	1.20543	1.60161	2.02779
0.3	45	0	0.26511	0.55046	0.87771	1.26812	1.73217	2.25038
0.3	60	0	0.26588	0.55739	0.90574	1.35193	1.93879	2.65684
0.3	75	0	0.26646	0.56278	0.92969	1.43759	2.22876	3.49853
0.3	90	0	0.26667	0.56483	0.93938	1.47789	2.43581	∞
0.4	0	0	0.26420	0.54291	0.85084	1.20098	1.59794	2.02789
0.4	15	0	0.26440	0.54454	0.85641	1.21419	1.62298	2.06774
0.4	30	0	0.26495	0.54912	0.87262	1.25419	1.70165	2.19629
0.4	45	0	0.26572	0.55573	0.89756	1.32117	1.84537	2.44683
0.4	60	0	0.26650	0.56278	0.92670	1.41098	2.07413	2.90761
0.4	75	0	0.26708	0.56827	0.95162	1.50309	2.39775	3.87214
0.4	90	0	0.26729	0.57035	0.96171	1.54653	2.63052	∞
0.5	0	0	0.26481	0.54814	0.87042	1.25310	1.70919	2.22144
0.5	15	0	0.26501	0.54980	0.87621	1.26726	1.73695	2.26685
0.5	30	0	0.26557	0.55447	0.89307	1.31017	1.82433	2.41367
0.5	45	0	0.26634	0.56119	0.91902	1.38218	1.98464	2.70129
0.5	60	0	0.26712	0.56837	0.94939	1.47906	2.24155	3.23477
0.5	75	0	0.26770	0.57394	0.97538	1.57881	2.60846	4.36620
0.5	90	0	0.26792	0.57606	0.98591	1.62599	2.87468	∞
0.6	0	0	0.26543	0.55357	0.89167	1.31379	1.85002	2.48365
0.6	15	0	0.26563	0.55525	0.89770	1.32907	1.88131	2.53677
0.6	30	0	0.26619	0.56000	0.91527	1.37544	1.98005	2.70905
0.6	45	0	0.26696	0.56684	0.94235	1.45347	2.16210	3.04862
0.6	60	0	0.26775	0.57414	0.97406	1.55884	2.45623	3.68509
0.6	75	0	0.26833	0.57982	1.00123	1.66780	2.88113	5.05734
0.6	90	0	0.26855	0.58198	1.01225	1.71951	3.19278	∞
			$\left[\begin{smallmatrix} (-5)5 \\ 4 \end{smallmatrix} \right]$	$\left[\begin{smallmatrix} (-4)4 \\ 6 \end{smallmatrix} \right]$	$\left[\begin{smallmatrix} (-3)2 \\ 7 \end{smallmatrix} \right]$	$\left[\begin{smallmatrix} (-3)7 \\ 7 \end{smallmatrix} \right]$		

See Examples 15-20.

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$$\Pi(n; \varphi|\alpha) = \int_0^\varphi (1 - n \sin^2 \theta)^{-1} [1 - \sin^2 \alpha \sin^2 \theta]^{-\frac{1}{2}} d\theta$$

n	$\alpha \backslash \varphi$	0°	15°	30°	45°	60°	75°	90°
0.7	0°	0	0.26605	0.55918	0.91487	1.38587	2.03720	2.86787
0.7	15	0	0.26625	0.56090	0.92116	1.40251	2.07333	2.93263
0.7	30	0	0.26681	0.56573	0.93952	1.45309	2.18765	3.14339
0.7	45	0	0.26759	0.57270	0.96784	1.53846	2.39973	3.56210
0.7	60	0	0.26838	0.58014	1.00104	1.65425	2.74586	4.35751
0.7	75	0	0.26897	0.58592	1.02954	1.77459	3.25315	6.11030
0.7	90	0	0.26918	0.58812	1.04110	1.83192	3.63042	∞
0.8	0	0	0.26668	0.56501	0.94034	1.47370	2.30538	3.51240
0.8	15	0	0.26688	0.56676	0.94694	1.49205	2.34868	3.59733
0.8	30	0	0.26745	0.57168	0.96618	1.54790	2.48618	3.87507
0.8	45	0	0.26823	0.57877	0.99588	1.64250	2.74328	4.43274
0.8	60	0	0.26902	0.58635	1.03076	1.77145	3.16844	5.51206
0.8	75	0	0.26961	0.59225	1.06073	1.90629	3.80370	7.96669
0.8	90	0	0.26982	0.59449	1.07290	1.97080	4.28518	∞
0.9	0	0	0.26731	0.57106	0.96853	1.58459	2.74439	4.96729
0.9	15	0	0.26752	0.57284	0.97547	1.60515	2.79990	5.09958
0.9	30	0	0.26808	0.57785	0.99569	1.66788	2.97710	5.53551
0.9	45	0	0.26887	0.58508	1.02695	1.77453	3.31210	6.42557
0.9	60	0	0.26966	0.59281	1.06372	1.92081	3.87661	8.20086
0.9	75	0	0.27025	0.59882	1.09535	2.07487	4.74432	12.46407
0.9	90	0	0.27047	0.60110	1.10821	2.14899	5.42125	∞
1.0	0	0	0.26795	0.57735	1.00000	1.73205	3.73205	∞
1.0	15	0	0.26816	0.57916	1.00731	1.75565	3.81655	∞
1.0	30	0	0.26872	0.58428	1.02866	1.82781	4.08864	∞
1.0	45	0	0.26951	0.59165	1.06170	1.95114	4.61280	∞
1.0	60	0	0.27031	0.59953	1.10060	2.12160	5.52554	∞
1.0	75	0	0.27090	0.60566	1.13414	2.30276	7.00372	∞
1.0	90	0	0.27112	0.60799	1.14779	2.39053	8.22356	∞
			$\left[\begin{smallmatrix} (-5)5 \\ 4 \end{smallmatrix} \right]$	$\left[\begin{smallmatrix} (-4)5 \\ 6 \end{smallmatrix} \right]$	$\left[\begin{smallmatrix} (-3)2 \\ 7 \end{smallmatrix} \right]$	$\left[\begin{smallmatrix} (-2)1 \\ 7 \end{smallmatrix} \right]$		